BC Multiple Choice Chapter 9

1969

30. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) $\sin x$ (B) $\cos x$ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

32. For what values of x does the series $1+2^x+3^x+4^x+\cdots+n^x+\cdots$ converge?

(A) No values of x (B) $x<-1$ (C) $x\geq -1$ (D) $x>-1$ (E) All values of x

45. The complete interval of convergence of the series $\sum_{k=1}^{\infty}\frac{(x+1)^k}{k^2}$ is

(A) $0<x<2$ (B) $0\leq x\leq 2$ (C) $-2<x\leq 0$

(D) $-2\leq x<0$ (E) $-2\leq x\leq 0$

1973

16. A series expansion of $\frac{\sin t}{t}$ is

(A) $1-\frac{t^2}{3!}+\frac{t^4}{5!}-\frac{t^6}{7!}+\cdots$

(B) $\frac{1}{t}-\frac{t}{2!}+\frac{t^3}{4!}-\frac{t^5}{6!}+\cdots$

(C) $1+\frac{t^2}{3!}+\frac{t^4}{5!}+\frac{t^6}{7!}+\cdots$

(D) $\frac{1}{t}+\frac{t}{2!}+\frac{t^3}{4!}+\frac{t^5}{6!}+\cdots$

(E) $t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+\cdots$

19. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{1}{n^2}$ II. $\sum_{n=1}^{\infty}\frac{1}{n}$ III. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$

(A) I only (B) III only (C) I and II only (D) I and III only (E) I, II, and III

1985

10. For $-1<x<1$ if $f(x)=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^{2n-1}}{2n-1}$, then $f'(x)=$

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$

(B) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n-2}$

(C) $\sum_{n=1}^{\infty}(-1)^{2n}x^{2n}$

(D) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n}$

(E) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n}$

14. Which of the following series are convergent?

I. $1+\frac{1}{2^2}+\frac{1}{3^2}+\ldots+\frac{1}{n^2}+\ldots$

II. $1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}+\ldots$

III. $1-\frac{1}{3}+\frac{1}{3^2}-\ldots+\frac{(-1)^{n+1}}{3^{n-1}}+\ldots$

(A) I only (B) III only (C) I and III only (D) II and III only (E) I, II, and III

31. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-1)^n}{n}$ converges?

(A) $-1\leq x<1$ (B) $-1\leq x\leq 1$ (C) $0<x<2$ (D) $0\leq x<2$ (E) $0\leq x\leq 2$

1988

13. $\sin(2x) =$

(A) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + \frac{(-1)^{n-1}x^{2n-1}}{(2n-1)!} + \ldots$

(B) $2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \ldots + \frac{(-1)^{n-1}(2x)^{2n-1}}{(2n-1)!} + \ldots$

(C) $-\frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \ldots + \frac{(-1)^n(2x)^{2n}}{(2n)!} + \ldots$

(D) $\frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots$

(E) $2x + \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} + \ldots + \frac{(2x)^{2n-1}}{(2n-1)!} + \ldots$

30. $\sum_{i=n}^{\infty} \left(\frac{1}{3}\right)^i =$

(A) $\frac{3}{2} - \left(\frac{1}{3}\right)^n$ (B) $\frac{3}{2}\left[1 - \left(\frac{1}{3}\right)^n\right]$ (C) $\frac{3}{2}\left(\frac{1}{3}\right)^n$

(D) $\frac{2}{3}\left(\frac{1}{3}\right)^n$ (E) $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

38. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges?

(A) $-1 \le x \le 1$ (B) $-1 < x \le 1$ (C) $-1 \le x < 1$

(D) $-1 < x < 1$ (E) All real x

44. Which of the following series converge?

I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{3}{2}\right)^n$

III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

1993

16. Which of the following series diverge?

I. $\sum_{k=3}^{\infty} \frac{2}{k^2+1}$

II. $\sum_{k=1}^{\infty} \left(\frac{6}{7}\right)^k$

III. $\sum_{k=2}^{\infty} \frac{(-1)^k}{k}$

(A) None (B) II only (C) III only (D) I and III (E) II and III

27. The interval of convergence of $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(A) $-3 < x \le 3$ (B) $-3 \le x \le 3$ (C) $-2 < x < 4$

(D) $-2 \le x < 4$ (E) $0 \le x \le 2$

43. The coefficient of x^6 in the Taylor series expansion about $x=0$ for $f(x)=\sin\left(x^2\right)$ is

(A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1

1997

14. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1,024}+\ldots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

17. Let f be the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$

(C) $(x-2)+(x-2)^2+(x-2)^3$

(D) $(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

(E) $(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$

20. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n\cdot 3^n}$ converges?

(A) $-3\le x\le 3$
(B) $-3<x<3$
(C) $-1<x\le 5$
(D) $-1\le x\le 5$
(E) $-1\le x<5$

24. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots$. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

76. Which of the following sequences converge?

I. $\left\{\frac{5n}{2n-1}\right\}$

II. $\left\{\frac{e^n}{n}\right\}$

III. $\left\{\frac{e^n}{1+e^n}\right\}$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

1998

14. What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$

(B) $1-\frac{1}{2}+\frac{1}{4}$

(C) $1-\frac{1}{3}+\frac{1}{5}$

(D) $1-\frac{1}{4}+\frac{1}{8}$

(E) $1-\frac{1}{6}+\frac{1}{120}$

18. Which of the following series converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(A) None
(B) II only
(C) III only
(D) I and II only
(E) I and III only

27. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n{}^{n-1}$

76. For what integer k, $k>1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

83. The Taylor series for $\ln x$, centered at $x=1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. Let f be the function given by the sum of the first three nonzero terms of this series. The maximum value of $|\ln x - f(x)|$ for $0.3 \le x \le 1.7$ is

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

84. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

89. The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots + \frac{(-1)^n x^n}{n!} + \ldots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

1969 BC

1. C
2. E
3. B
4. D
5. E
6. B
7. D
8. C
9. D
10. A
11. B
12. E
13. C
14. D
15. B
16. B
17. B
18. E
19. C
20. A
21. B
22. E
23. D
24. C
25. A
26. C
27. C
28. D
29. C
30. D
31. C
32. B
33. A
34. D
35. A
36. B
37. D
38. A
39. D
40. E
41. D
42. B
43. E
44. E
45. E

1973 BC

1. A
2. D
3. A
4. C
5. B
6. D
7. D
8. B
9. A
10. A
11. E
12. D
13. D
14. A
15. C
16. A
17. C
18. D
19. D
20. E
21. B
22. C
23. C
24. A
25. B
26. D
27. E
28. C
29. A
30. B
31. E
32. C
33. A
34. C
35. C
36. E
37. E
38. B
39. D
40. C
41. D
42. D
43. E
44. A
45. E

1985 BC

1. D
2. A
3. B
4. D
5. D
6. E
7. A
8. C
9. B
10. A
11. A
12. A
13. B
14. C
15. C
16. C
17. B
18. C
19. D
20. C
21. B
22. A
23. C
24. D
25. C
26. E
27. E
28. E
29. D
30. B
31. D
32. E
33. C
34. A
35. B
36. E
37. A
38. C
39. A
40. A
41. C
42. E
43. E
44. A
45. D

1988 BC

1. A
2. D
3. B
4. E
5. C
6. C
7. A
8. A
9. D
10. D
11. A
12. B
13. B
14. A
15. E
16. A
17. D
18. E
19. B
20. E
21. D
22. E
23. E
24. D
25. D
26. C
27. B
28. E
29. B
30. C
31. C
32. E
33. E
34. C
35. A
36. E or D
37. D
38. C
39. C
40. E
41. B
42. A
43. A
44. A
45. B

1993 BC

1. A
2. C
3. E
4. B
5. D
6. A
7. A
8. B
9. D
10. E
11. E
12. E
13. C
14. B
15. D
16. A
17. A
18. B
19. B
20. E
21. A
22. B
23. D
24. C
25. D
26. B
27. C
28. A
29. E
30. C
31. A
32. B
33. A
34. E
35. A
36. E
37. B
38. C
39. C
40. C
41. C
42. E
43. A
44. E
45. D

1997 BC

1. C
2. E
3. A
4. C
5. C
6. A
7. C
8. E
9. A
10. B
11. C
12. A
13. B
14. C
15. D
16. B
17. B
18. C
19. D
20. E
21. A
22. C
23. E
24. D
25. A
76. D
77. E
78. A
79. D
80. B
81. D
82. B
83. E
84. C
85. D
86. A
87. B
88. C
89. D
90. B

1998 BC

1. C
2. A
3. D
4. A
5. A
6. E
7. E
8. B
9. D
10. E
11. A
12. E
13. B
14. E
15. B
16. C
17. D
18. B
19. D
20. E
21. C
22. A
23. E
24. C
25. C
26. E
27. D
28. C
76. D
77. E
78. B
79. A
80. B
81. B
82. B
83. C
84. B
85. C
86. C
87. D
88. C
89. A
90. A
91. E
92. D